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The Strength of Nodule-Decorated Whiskers

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In this work, the strength of whisker containing nodules was studied. The relationship between the nodule geometry and the strength of the whisker is reported. The optimum ratios of the nodule radius to the whisker radius and the distance between two nodules to the whisker radius are presented.

An increase in the nodule radius can increase the strength of the whisker-reinforced composite. Theoretical calculations indicate that, when the radius ratio of the nodule to the whisker is 2 and when the ratio of the distance between two nodules to the whisker radius is in the range of 2 to 2.5, whisker strength reaches an optimum value. The selection of the ratios is affected by a number of factors, such as the ratio of the distance between two nodules to the whisker radius and the stress distribution in the whisker and the nodule. This study provides practical design values for the fabrication of whiskers.

1. Introduction

DURING the last few decades, a considerable amount of research work has been performed on reinforced composite materials. This can be attributed to the high strength and low density of composite materials compared to conventional engineering materials. Although there are a wide variety of reinforcing agents, whiskers have unique properties. Whiskers exhibit high mechanical strength and have diameters on the order of a micron.[1] These two factors enable whiskers to be excellent materials for reinforcing metals, polymers, and ceramics.

Whiskers are strong because they are essentially perfect crystals, and their extremely small diameters do not permit defects, which weaken larger crystals. In comparing whiskers to conventional materials, the latter contain a multiplicity of grain boundaries, voids, dislocations, and imperfections, whereas the single-crystal whisker, which approaches structural perfection, has none of these defects.

In general, whiskers have very large length-to-diameter (l/D) ratios of about 10 to 10,000. Previous work[2] has shown that as the diameter of the whisker becomes smaller its strength increases rapidly. From a mechanical point of view, a decrease in the size of the material will reduce the possibility of the production of a flaw. For cylindrical reinforcing agents of a given volume, a decrease in the diameter of the whisker will increase the surface area of the reinforcing agent. This results in an increase in the interfacial area per unit volume between the reinforcing agent and the matrix. Thus, the strength of the composite will increase.

In general, the whiskers that have previously been studied have been cylindrical or needle-shaped, because these are the type of whiskers that are grown by conventional production methods. It would be expected that the geometry of the whisker would strongly affect the strength of the whisker-reinforced composite. In 1963, W.F. Knippenberg[3] first produced whisker containing nodules. More recently, these types of whiskers have been produced by the carbothermic reduction of silica in a plasma furnace.[4] A scanning electron micrograph of the whiskers is shown in Fig. 1. The existence of nodules along the whisker not only increases the surface area of the whisker, but also increases the shear strength between the whisker and the matrix, and thus the strength of the whisker-reinforced composite.

In this article, the theoretical relationship between the nodule geometry and the strength of the whisker is determined. Also, the effect of factors, such as the ratio of the nodule radius to the whisker radius and the ratio of the distance between two nodules to the whisker radius, on the strength and the surface area of the whisker are discussed. In addition, the optimum ratios of the nodule radius to the whisker radius and the distance between two nodules to the whisker radius are presented.

2. Theoretical Considerations

The main application of whiskers is as a reinforcing agent for metal, ceramic, and polymer composites. In a composite, the whisker carries the majority of the load. Consider a single whisker containing nodules that are uniformly distributed

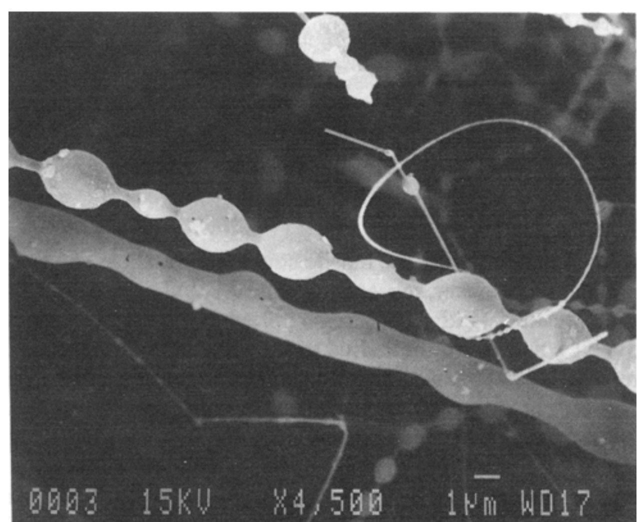

Fig. 1 A scanning electron micrograph of silicon carbide whiskers containing nodules.

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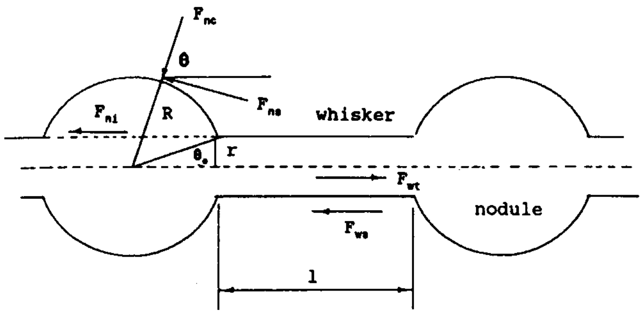

Fig. 2 Schematic diagram of a whisker containing nodules.

along the whisker in a composite, as shown in Fig. 2. When a force, P, is applied to the whisker, the whisker between two nodules is subjected to a tensile force F_{wt} and a shear force F_{ws}. The nodule is subjected to a compressive force, F_{nc} and a shear force F_{ns}. Thus, the total force balance is:

$$\Sigma = F_{wt} - F_{ws} - F_{ns} \cos \theta - F_{ns} \sin \theta = 0 \quad [1]$$

where θ is the angle between the applied force acting on the whisker and the compressive force acting on the nodule. Because

$$F_{wt} = \sigma_{wt} A_{w,}\ F_{ws} = \tau_{ws} S_{w,}\ F_{nc} \cos \theta = \int \sigma \cos \theta \, ds,$$

and

$$F_{ns} \sin \theta = \int \tau_{ns} \sin \theta \, ds,$$

Eq 1 can also be expressed in the following form:

$$\sigma_{wt} A_w - \tau_{ws} S_w - \int \sigma_{nc} \cos \theta \, ds - \int \tau_{ns} \sin \theta \, ds = 0 \quad [2]$$

Integrating this yields:

$$2\sigma_{wt} \left(\frac{r}{R}\right)^2 - 4\tau_{wt} \left(\frac{rl}{R^2}\right) - \sigma_{nc} \left(\pi - \sin 2\theta_o - 2\theta_o\right) - \tau_{ns} \left(1 + \cos 2\theta_o\right) = 0 \quad [3]$$

where σ_{wt} and τ_{wt} are the tensile stress and the shear stress applied to the whisker, respectively, and σ_{nc} and τ_{ns} are the compressive stress and shear stress applied to the nodule, respectively; r and R are the radius of the whisker and the nodule, respectively; A_w and S_w are the cross-sectional area and the surface area of the whisker between two nodules, respectively, and l is the distance between two nodules, and $\theta_o = \arcsin(r/R)$.

In general, failure of the whisker containing nodules in a composite can occur by three different mechanisms. Fracture at the cross section of the whisker is a common failure mode. This type of failure can be found in polymer, ceramic, and metal matrix composites. The second mechanism is fracture at the interface between the nodule and the whisker. This is likely when the whisker/nodule interface is weak in comparison to the inherent strength of the whisker. The third failure is cracking of the nodule. The last two phenomena would only be observed in metal and ceramic matrix composites where the matrix would be strong enough to carry the large destructive forces.

2.1 *The Strength of the Whisker*

When the strength of the whisker is lower than that of either the interface between the nodule and the whisker, or the nodule itself, failure may take place along the cross section of the whisker. In the first case:

$$F_{wt} \leq F_{nc} \cos \theta + F_{ns} \sin \theta \quad [4]$$

Therefore:

$$\sigma_{wt} A_w \leq \int \sigma_{nc} \cos \theta \, ds + \int \tau_{ns} \sin \theta \, ds \quad [5]$$

Integrating Eq 5:

$$2\sigma_{wt} \left(\frac{r}{R}\right)^2 \leq \sigma_{nc} \left(\pi - \sin 2\theta_o - 2\theta_o\right) + \tau_{ns} \left(1 + \cos 2\theta_o\right) \quad [6]$$

In the second case:

$$F_{wt} \leq F_{ni} \quad [7]$$

Therefore:

$$\sigma_{wt} r^2 \pi \leq \tau_{ni} 2\pi r \left[2\left(R^2 - r^2\right)^{1/2}\right] \quad [8]$$

Simplifying Eq 8,

$$\sigma_{wt} \leq 4\tau_{ni} \left[\left(\frac{R}{r}\right)^2 - 1\right]^{1/2} \quad [9]$$

2.2 *Strength of the Interface Between the Nodule and the Whisker*

It can be seen from Fig. 2 that, when a force is applied to the whisker, the nodule is subjected to a compressive stress and a shear force, whereas the whisker itself is under a tensile stress. The sum of these stresses is a shear force, F_{ni}, which occurs at the interface between the nodule and the whisker. Fracture will occur at this position when the strength of the interface is lower than that of both the whisker and the nodule. In the first case:

$$F_{ni} \leq F_{wt} \quad [10]$$

Therefore:

$$\tau_{ni} S_{ni} \leq \sigma_{wt} A_w \quad [11]$$

where τ_{ni} is the shear stress at the interface between the nodule and the whisker, and S_{ni} is the interfacial area between the nodule and the whisker.

Substituting and rearranging Eq 11:

$$\frac{\sigma_{wt}}{\tau_{ni}} \geq 4\left[\left(\frac{R}{r}\right)^{2} - 1\right]^{1/2} \qquad [12]$$

For the second case:

$$F_{ni} \leq F_{nc} + F_{ns} \qquad [13]$$

Therefore:

$$\tau_{ni} 2\pi r\, [\{2 \left(R^2 - r^2\right)^{1/2} \leq \frac{\pi}{2}R^2 \left[\sigma_{nc}\left(\pi - \sin 2\theta_o - 2\theta_o\right)\right.$$
$$\left.\left[+ \tau_{ns}\left(1 + \cos 2\theta_o\right)\right]\right] \qquad [14]$$

Simplifying:

$$8\frac{r}{R}\left[1 - \left(\frac{r}{R}\right)^{2}\right]^{1/2} \leq \frac{\sigma_{nc}}{\tau_{ni}}\left(\pi - \sin 2\theta_o - 2\theta_o\right)$$
$$+ \frac{\tau_{ns}}{\tau_{ni}}\left(1 + \cos 2\theta_o\right) \qquad [15]$$

2.3 *Strength of the Nodule*

When the whisker is used as a reinforcing agent in the ceramic or metal matrix composite, the nodule can crack prior to the fracture of either the whisker or the interface between the nodule and the whisker. In the first case:

$$F_{nc} \cos\theta + F_{ns} \sin\theta \leq F_{wt} \qquad [16]$$

Thus:

$$\frac{\pi}{2}R^2 \left[\sigma_{nc}\left(\pi - \sin 2\theta_o - 2\theta_o\right)\right.$$
$$\left.\left[+ \tau_{ns}\left(1 + \cos 2\theta_o\right)\right]\right] \leq \sigma_{wt} r^2 \pi \qquad [17]$$

Simplifying:

$$\sigma_{nc}\left(\pi - \sin 2\theta_o - 2\theta_o\right)$$
$$+ \tau_{ns}\left(1 + \cos 2\theta_o\right) \leq 2\sigma_{wt}\left(\frac{r}{R}\right)^2 \qquad [18]$$

For the second case:

$$F_{nc} \cos\theta + F_{ns} \sin\theta \leq F_{ni} \qquad [19]$$

Thus:

$$\frac{\pi}{2}R^2 \left[\sigma_{nc}\left(\pi - \sin 2\theta_o - 2\theta_o\right)\right.$$
$$\left.\left[+ \tau_{ns}\left(1 + \cos 2\theta_o\right)\right]\right]$$
$$\leq \tau_{ni} 2\pi r l \left[2\left(R^2 - r^2\right)^{1/2}\right] \qquad [20]$$

Simplifying:

$$\sigma_{nc}\left(\pi - \sin 2\theta_o - 2\theta_o\right) + \tau_{ns}\left(1 + \cos 2\theta_o\right)$$
$$\leq 8\tau_{ni}\left(\frac{r}{R}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]^{1/2} \qquad [21]$$

Equations 3, 6, 9, 13, 15, 18, and 21 provide theoretical relationships between the geometry of the whisker containing nodules and the mechanical behavior of the whisker under a constant applied force. It can be seen that when a whisker-containing nodule is subjected to a force, the strength of the

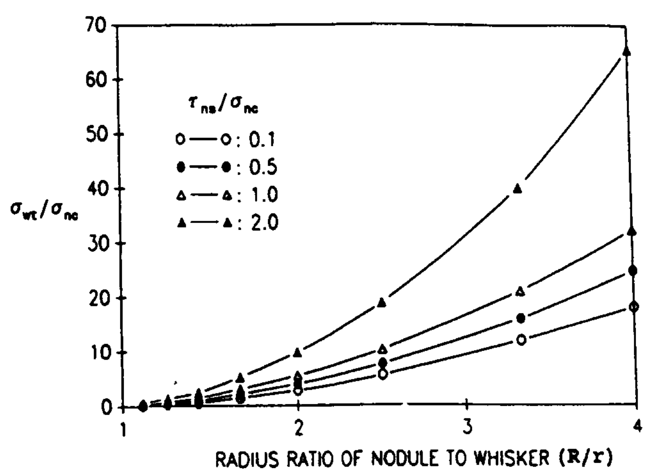

Fig. 3 The effect of the radius ratio of nodule to whisker (R/r) on the ratio of the tensile stress on the whisker to the compressive stress on the nodule (σ_{wt}/σ_{nc}) at various ratios of the shear stress to the compressive stress on the nodule (τ_{ns}/σ_{nc}).

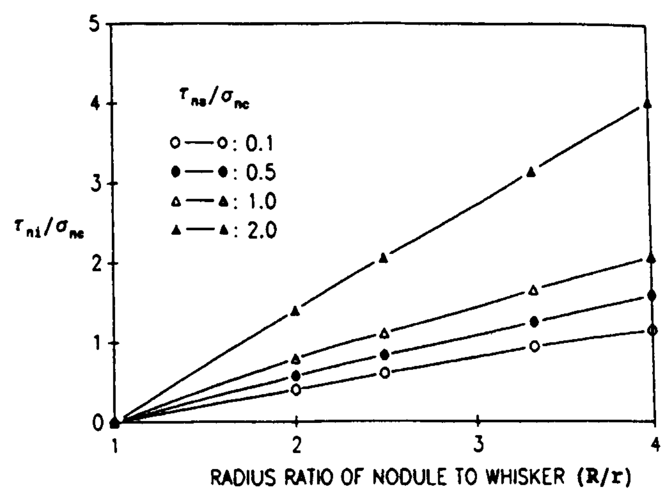

Fig. 4 The effect of the radius ratio of the nodule to the whisker (R/r) on the ratio of the shear stress at the interface between the nodule and the whisker to the compressive stress on the nodule (τ_{ni}/σ_{nc}) at various ratios of the shear stress to the compressive stress on the nodule (τ_{ns}/σ_{nc}).

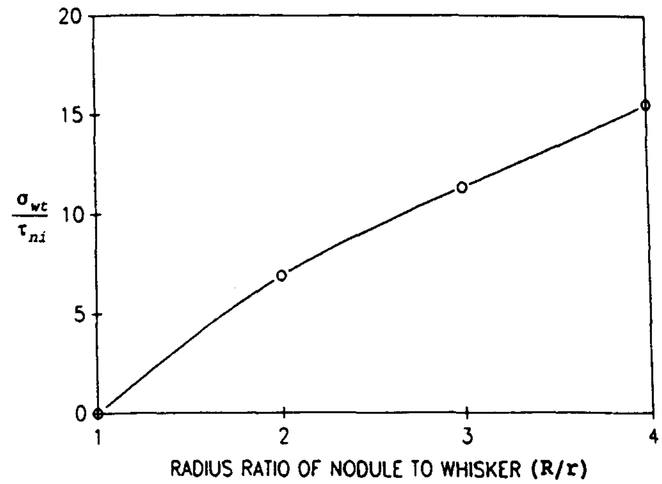

Fig. 5 The effect of the radius ratio of the nodule to the whisker (R/r) on the ratio of the tensile stress on the whisker to the shear stress at the interface between the nodule and the whisker σ_{wt}/τ_{ni}.

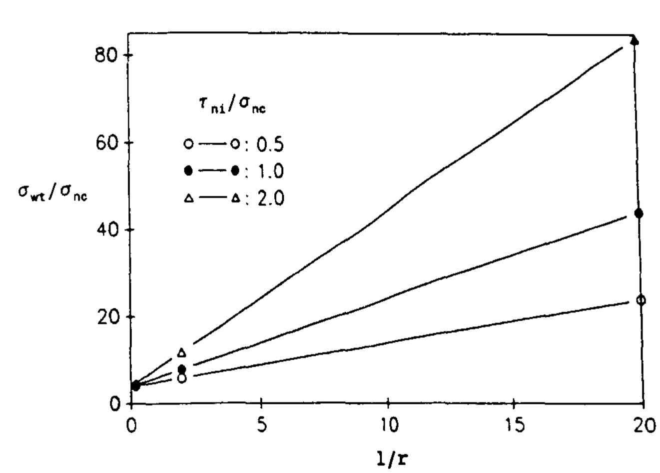

Fig. 6 The effect of the ratio of the distance between two nodules to the whisker radius (l/r) on the ratio of the tensile stress on the whisker to the compressive stress on the nodule (σ_{wt}/σ_{nc}) at various ratios of the shear stress on the whisker to the compressive stress on the nodule (τ_{wt}/σ_{nc}).

whisker is strongly affected by the tensile and shear stresses acting on the whisker and the nodules, the radii of the nodule and the whisker, and the distance between the two nodules (*i.e.*, the distribution density of the nodules along the whisker).

3. Discussion

The mechanical strength of a whisker containing nodules in a composite depends not only on its inherent properties, but also on its geometry. The geometry of the whisker can strongly affect the stress distribution in the whisker and the nodule. Based on the relationships shown in Eq 3, 6, 9, and 15, the geometric factors that affect the mechanical properties of the whisker, such as the nodule radius, whisker radius, and the distance between two nodules, will be discussed below. In addition, the effect of the geometry of the whisker on the interfacial area will also be discussed.

3.1 *Effect of the Ratio of Nodule Radius to Whisker Radius*

The effect of the ratio of nodule radius to the whisker radius, (R/r), on the stress ratio of the nodule to the whisker is shown in Fig. 3. As the ratio of the nodule radius in the whisker radius increases from 1 to 2, the ratio of stress in the nodule to the stress in the whisker increases slightly. Under these conditions, σ_{wt}/σ_{nc}, the ratio of the tensile stress applied in the whisker to the compressive stress applied to the nodule is in the range of 2.9 to 9.7, for τ_{ns}/σ_{nc} ratios in the range of 0.1 to 2. This result means that the stress is relatively uniformly distributed to the whisker and the nodule. However, when the radius ratio of the nodule to the whisker increases from 2 to 4, the ratio of σ_{wt}/σ_{nc} increases dramatically, *e.g.*, σ_{wt}/σ_{nc} increases from 2.76 to 18, when $\tau_{ns}/\sigma_{nc} = 0.5$, and σ_{wt}/σ_{nc} increases from 9.7 to 65.6 when $\tau_{ns}/\sigma_{nc} = 2.0$. This result indicates that, when the radius ratio is greater than 2, the stress is concentrated in the whisker rather than in the nodule. In this case, when a force is applied to a composite, the whisker will fail prior to the nodule.

The effect of the radius ratio of the nodule to the whisker on the stress distribution at the interface between the nodule and the whisker is shown in Fig. 4. An increase in the ratio of the nodule radius to the whisker radius will result in an increase in the ratio of τ_{ni}/σ_{nc}, that is either the shear stress increases at the interface or the compressive stress decreases in the nodule. The ratio of τ_{ns}/σ_{nc} increases with the ration of τ_{ni}/σ_{nc} at a given radius ratio of nodule to whisker. This results indicates that the stress at the interface increases with increasing radius ratio, R/r. This result supports the results shown in Fig. 3.

The effect of the radius ratio of the whisker to the nodule on the stress distribution in the whisker and at the interface is shown in Fig. 5. An increase in the radius ratio of the nodule to the whisker can cause either an increase in the tensile stress, σ_{wt}, in the whisker, or a decrease in the shear stress, tni, at the interface between the nodule and the whisker.

Based on the above results, it can be seen that for optimum properties, the ratio of nodule radius to whisker radius should be about 2. It would be expected that the strength of the whisker containing nodules would be higher than that of a cylindrical whisker of the same radius, because the cross-sectional area of the nodule and the area resisting shear is larger than that of an undecorated whisker. An increase in the nodule radius can increase the strength of the whisker containing nodules. On the other hand, if the ratio of the nodule radius to the whisker radius exceeds 2, the stress is concentrated largely in the whisker, and the strength of the whisker cannot attain the maximum value. The selection of an optimum ratio of the nodule radius to the whisker radius would depend on the individual situation, such as the distribution of the tensile and the shear stresses in the whisker and the distance between two nodules. The maximum strength of the whisker is achieved when the ratio of the nodule radius to the whisker radius is 2.

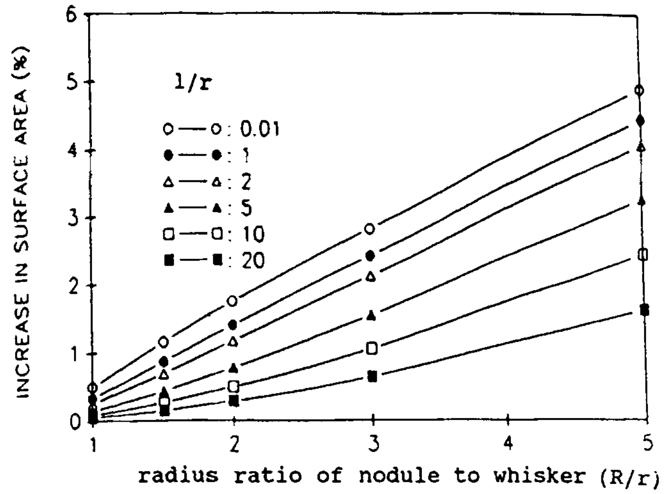

Fig. 7 The effect of the radius ratio of the nodule to the whisker (R/r) on the increase of the surface area of the whisker at various ratios of the distance between two nodules (l/r).

3.2 *Effect of the Distance Between Two Nodules*

The effect of the distance between two nodules on the strength of the whisker is illustrated in Fig. 6. From the relationships shown in Eq 3, 6, 9, and 15, it can be seen that the factors that affect the mechanical properties of whisker containing nodules are related to each other. The results shown in Fig. 6 are based on the condition that the nodule radius is twice that of the whisker radius, that is $R/r = 2$. The ratio of σ_{wt}/σ_{nc} increases linearly with l/r, which is the ratio of the distance between two nodules to the whisker radius. The ratio of whisker shear stress to the nodule compressive stress, τ_{wt}/σ_{nc}, has a strong effect on the ratio σ_{wt}/σ_{nc}. For example, when $\tau_{wt}/\sigma_{nc} = 0.5$, l/r increases from 0.2 to 20, whereas σ_{wt}/σ_{nc} increases from 4.16 to 23.96. However, when $\tau_{wt}/\sigma_{nc} = 2$, l/r increases from 0.2 to 20, and σ_{wt}/σ_{nc} increases by about 80, that is from 4.76 to 83.96. Therefore, either an increase in the number of nodules along the whisker, or a decrease in the distance between two nodules, will result in an increase in the strength of the whisker. However, if the distance between two nodules is too small, then there will be a sharp change in the whisker profile. This will result in a high stress concentration and will lower the strength of the whisker. Mechanical theory[5] shows that the stress concentration factor, $K_f = \sigma_{max}/\sigma_{nor}$, which is the ratio of the maximum stress due to the stress concentration to the normal stress, is much higher when there is a sharp corner in the material. It is estimated that, when r/R is in the range of 0.08 to 0.5, K_f increases from 1.4 to 2.4. When r/R is above 0.5, K_f is less than 1.4—that is, when the radius ratio of the nodule to the whisker is above 0.5, then the stress concentration due to the sharp change in the profile of the whisker becomes very small. Therefore, it is recommended that the ratio of the distance between two nodules to the whisker radius should be in the range of 2 to 2.5. If the ratio is lower than 2, a high stress concentration will be developed in the area between the nodules. If the ratio is too high, a high tensile stress will be developed in the whisker between the nodules. This will allow further loading of the strong whiskers beyond that which is possible with whiskers that do not have nodules, or that have nodules that are too close. This will weaken the whisker.

3.3 *Surface Area of Whisker Containing Nodules*

As a reinforcing agent, the interfacial strength of the whisker is particularly important in a composite. The mechanical strength of a composite material depends not only on the properties of the reinforcing agent and the matrix phases, but also on the strength of the interface between the reinforcing phase and the matrix. Increasing the interfacial area results in improved bonding between the whisker and the matrix and, consequently, an increase in the strength of the composite.

The increase in surface area due to the presence of nodules in the whiskers can be calculated as follows:

$$\Delta S = \frac{S'_w}{S_w} \qquad [22]$$

Substituting and simplifying:

$$Nn\frac{\left(2\pi rl + 4\pi R^2 - 2r^2\pi\right)}{N2\pi rL} = \frac{2\left(\frac{R}{r}\right)^2 - 1}{2\frac{R}{r} + \frac{l}{r}} \qquad [23]$$

where $S_w = N2\pi rL$ is the total surface area of a cylindrical whisker; $S'_w = n(2\pi rl + 4\pi R^2 - 2\pi r^2)$ is the surface area of a whisker containing nodules; $L = n\,2R + l)$ is the length of the cylindrical whisker; N is the number of whiskers in the composite; and n is the number of nodules existing in one single whisker.

The effect of nodule radius on the surface area of the whisker is shown in Fig. 7. These results show that the surface area of the whisker containing nodules increases smoothly and isotonically with increasing radius ratio of the nodule to the whisker. The surface area increases more rapidly for whiskers that have a small distance between nodules, *i.e.*, those that have a large number of nodules. A decrease in the ratio of the distance between two nodules to the whisker radius can increase the surface area of the whisker. This result shows that beyond a certain size the distribution density of the nodules and the area resisting shear becomes more than is needed to load up the stronger whiskers.

4. Conclusions

Whisker containing nodules are superior to ordinary whiskers as reinforcing agents. The existence of nodules along the whisker results not only in an increase in the surface area of the whisker, but also an increase in the shear strength between the whisker and the matrix and an increase in the stress carried by the whisker itself. The relationship between the whisker/nodule geometries and the strength of the whisker containing nodules has been investigated theoretically. The results show that the strength and surface area of the whisker containing nodules are strongly affected by the nodule radius, the whisker radius, and the distance between two nodules. Generally, whisker containing nodules exhibit the optimum composite properties when the ratio of nodule radius to whisker radius is 2 and when the ratio of the distance between two nodules to the whisker radius is in the range of 2 to 2.5.

Acknowledgment

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